B.Sc-III, paper-VI, group-A (Nuclear physics)

Synchrotron :-

For the non-relativistic motion of a charged particle in a cyclotron, K.E is given by

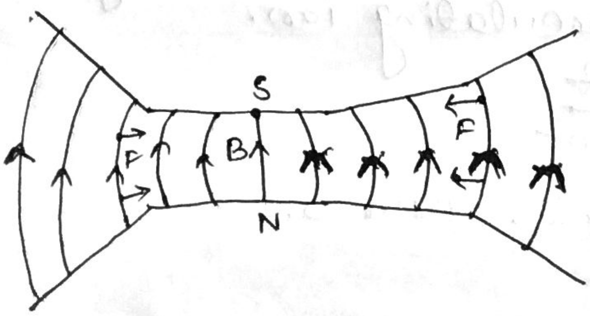
$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{B^2r^2q^2}{m} = qV \quad \text{--- (1)}$$

Where V is the effective potential difference to which the particles have been accelerated

$$V = \frac{1}{2}B^2r^2\frac{q}{m} \quad \text{--- (2)}$$

The above equation shows that the energy increases as the square of both B and r. As the mass m increases for a fixed B and q, the required frequency goes down and as a result the particles in a fixed frequency motion go out of phase which the accelerating voltage.

Another important point is that, the magnetic field must be shaped so that if a particle moves vertically away from a central plane, it experiences a restoring force. To accomplish this the magnetic force should get weaker with the increase of the radius. This is represented by curved lines of force. The force experienced by the particle above and below the central plane is also indicated.



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Thus, the vertical focussing requirements implies that the field must decrease with radius. This is Just opposite to the variation required to a compensate for relativistic increase in mass. It is clear from the equation $\frac{m\omega^2}{r} = Bqv$ that as m increases, the frequency ν can remain constant, if B increases to keep $\frac{B}{m}$ = Constant.

The above approach Contradicts the focussing Conditions of the beam. Relativity sets a limit to maximum energy that can be obtained using a cyclotron, which is ~22 MeV.

Calculation of fractional frequency shift :—

The maximum frequency of the ac field is adjusted to the value of ν near the ion source at the centre. At the centre, the K.E of the particle E_c is small and $B = B_c$. let E_K be the KE of the particle as it reaches the D's radius, where B is equal to $B_c - \Delta B$. let the frequency of the oscillator output be reduced to $f_c - \Delta f$, where f_c is the maximum frequency at the oscillator near the ion source at the centre, so that it is matched always between the frequency of ac voltage and that of circulating ion.

we have, $f_c = \frac{1}{2\pi} \cdot B_c \frac{qc^2}{m_0c^2}$

$\because$ $E_c << m_0c^2$, as the centre of the D.

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and, $$f_c - \Delta f = \frac{1}{2\pi}\left(\frac{B_c - \Delta B}{m_0c^2 + E_K}\right)qc^2 \quad \text{--- (3)}$$

$$\therefore E_K = \frac{qc^2}{2\pi}\cdot\frac{(B_c-\Delta B)}{(f_c-\Delta f)} - m_0c^2 = \frac{qc^2}{2\pi}\left[\frac{B_c-\Delta B}{f_c-\Delta f} - \frac{B_c}{f_c}\right] \text{ --}$$

$$= \frac{qc^2}{2\pi f_c^2}\left[B_c\,\Delta f - f_c\,\Delta f\right] = \frac{qc^2}{2\pi f_c}\left[\frac{\Delta f}{f_c} - \frac{\Delta B}{B_c}\right]$$

$$= m_0c^2\left[\frac{\Delta f}{f_c} - \frac{\Delta B}{B_c}\right]$$

$$\therefore \frac{\Delta f}{f_c} = \frac{E_K}{m_0c^2} + \frac{\Delta B_c}{B_c} \quad \text{--- (4)}$$

The above equation gives the fractional frequency shift required to give a final energy E_K to the charged particle.

The increase in energy ΔE at the D's crossing is

$$\Delta E = qV_0 \sin\phi_0 \quad \text{--- (5)}$$

where V_0 is the peak value of the accelerating voltage and ϕ_0 is the synchronous phase angle.

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